

# A single-bit narrow-band bandpass digital filter

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**SUMMARY:** *We provide a practical framework for filtering bit streams with a bandpass of exactly one bit. Although the output is a filtered single bit, this structure uses just the one multiplication arithmetic present in conventional filtering procedures. The filter incorporates an efficient ternary filter cascaded with a recursive sigma-delta filter, and this work expands upon the authors' prior work in which a low-pass ternary filter was constructed. The common thread throughout these efforts is the use of single-bit signal processing algorithms for effective and straightforward signal filtering. In this continuation of the topic, we offer a bandpass single-bit filter that combines a ternary finite-impulse-response (FIR) filter with a sigma-delta modulator that is optimized for bandpass operation. A sigma-delta modulator is required to remodulate the filtered signal back into the single-bit format after it has passed through a ternary filter, which yields a multi-bit output.*

## 1 INTRODUCTION;

Sigma-delta ADC results in an oversampled single-bit format, therefore conventional multi-bit output requires resampling to the Nyquist rate and filtering.<sup>1</sup> This filtering gets rid of the harsh, typically single-bit, quantization noise that was created. After the signal has been filtered, conventional DSP operations are applied to it.

The potential and advantages of single-bit processing have just lately been revealed.<sup>2,3,4,5</sup> Typically, sigma-delta modulator (m) bit-stream filtering is used for such reporting because of its promising efficiency and ease of implementation. Recently, however, a full single-bit QPSK demodulator was described in<sup>5</sup>, and single-bit digital signal processing was shown in<sup>4</sup>. Many of these single-bit methods may be easily implemented using a field-programmable-gate-array (FPGA).

The use of single-bit representations for processing signals has several benefits. The alphabet of the single-bit format is -1 and 1. This greatly simplifies the process of multiplying single-bit signals. A multiplexer may be used instead of a multiplier to do the multiplication. When compared to the multi-bit format, the single-bit format reduces the amount of internal signal routing needed in an FPGA or integrated circuit. Finally, the decimation and filtering stages are unnecessary in sigma-delta analog-to-digital converters, allowing for their simplification. **Many multiplier-free filtering designs are motivated, at least in part, by the decreased complexity available when multiplication is conducted in the single-bit format.**<sup>2,3</sup> **Because of the complexity of the circuitry required, multi-bit multiplications take up more space than their single-bit counterparts. The multiplication operator has been the subject of recent efforts to improve filtering efficiency. The elimination of FIR multiplication** Coarse quantization may be used for filtering in one of two ways:

Many recent studies and implementations of efficient filtering provide multi-bit filtered outputs. This is not an environment that encourages effective rollouts. Filters that maintain a single-bit resolution after filtering have received very little attention. A zero-padded FIR filter is used to process 3, 7, and 8 In3, m bit streams. After being created at the Nyquist rate, the filter coefficients are zero padded R times. The frequency response of the filter is replicated by the spectrum R times due to the zero padding. The usage of a single input bit maximizes efficiency. Therefore, the FIR filter's internal multiplication of taps and input signal is reduced to a straightforward multiplexing operation. Lowpass filtering and remodulation at single-bit resolution were applied to the spectral copies using a m. For single-bit inputs, this filter design shines, but multi-bit inputs still need complicated multiplication. In 7, a new, more effective method of filtering single bits was suggested. An IIR filter with a m is used to do filtering on a single bit. The symbol m functions as

the lowpass architecture of '8. Part 3 expands the discussion of the lowpass case to the bandpass situation. This paper presents a simplified construction for a bandpass single-bit filter. Finally, section 4 presents and discusses simulations of a prototype filter at various oversampling ratios.

## 2 THE LOWPASS FILTER WITH ONLY ONE BIT

Figure 1 depicts the layout of a basic low-pass filter that uses just one bit of information. A ternary FIR filter and a recursive filter are the building blocks of the system. The output of the recursive filter is remodulated to single-bit resolution using a  $m$  as a delay element.

An FIR filter with a smaller coefficient alphabet is the ternary filter. The coefficients (taps) of the ternary filter are limited to the values  $[-1]$ ,  $[0]$ , and  $[1]$ . Convolution of the ternary taps  $h(i)$  and the input signal  $x(k)$  is a mathematical description of the FIR filter output  $y(k)$ . If the FIR filter's order is  $M$ , then the ternary filter's output will also be  $M$ . the IIR filter's delay component. Although this structure speeds up implementation, it comes with significant downsides.

Non-linear effects are a common problem for IIR structures.  $y(k) \star$

$$\sum_{i=0}^M x(k-i)h(i), h(i) = 1, 0, -1$$

(1)

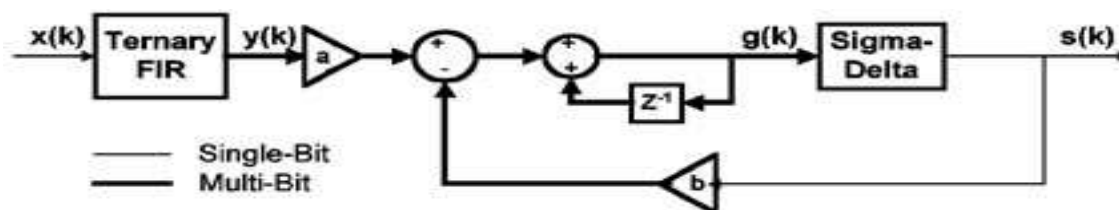
phase and are quite susceptible to coefficient quantization compared to FIR filters. A quasi-orthonormal structure was employed for higher orders than the first.<sup>7</sup> In this topology, bit-stream multiplication was also simplified by using  $m$ . In order to accomplish these high orders, numerous  $m$ 's are needed. While this streamlines the multiplication process, all the  $m$ 's create a lot of background noise.

The authors have previously shown a one-bit filter based on a recursive remodulating filter and an efficient ternary FIR filter.<sup>8,9</sup> Advantages of FIR filters across a small bandwidth, such as in<sup>3</sup>, are present in this filter, but the efficiency of an IIR filter, as in<sup>7</sup>, is also taken use of. But the single-bit filter in<sup>8</sup> can filter both multi-bit and single-bit signals effectively, unlike the work in<sup>3</sup>. References 9, 10, and 14 provide further information.

We then quickly introduce the single-bit

Figure 2 depicts the internal workings of a ternary FIR filter. Due of the restricted tap values, a straightforward hardware solution is possible. Simple logic gates or a lookup table may be used to do multiplication in the single-bit domain.<sup>6</sup> The zero in the ternary alphabet is implemented with a delay, and there is no need to provide any data to the ternary filter's final summer. The authors ran several simulations and discovered that for basic lowpass filters, up to half of the tap values might be zero. This makes hardware implementation of ternary filters easy and efficient, since the number of bits needed at the summer's output is greatly reduced.

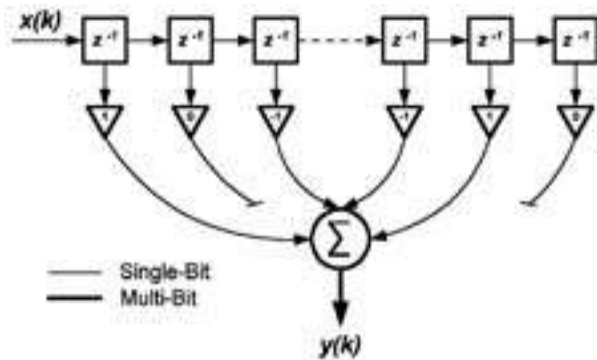
The authors propose creating the ternary taps using  $m$ , like that which was used in<sup>2</sup>. Although other approaches have been used to create ternary coefficients (6, 11), such methods are computationally highly costly since they employ things like dynamic programming and min-max search algorithms.



**Figure 1:** Block diagram of a single-bit lowpass filter.

"A single-bit narrow-band bandpass digital filter" - Thompson, Hussain & O'Shea

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**Figure 2:** Block diagram of a ternary FIR filter.

The modification of filter coefficients with  $m$  was studied in depth in [2]. The approach explains why ternary coefficients are preferable

influence on the SQNR. The results of this study served as the foundation for developing ternary coefficients.

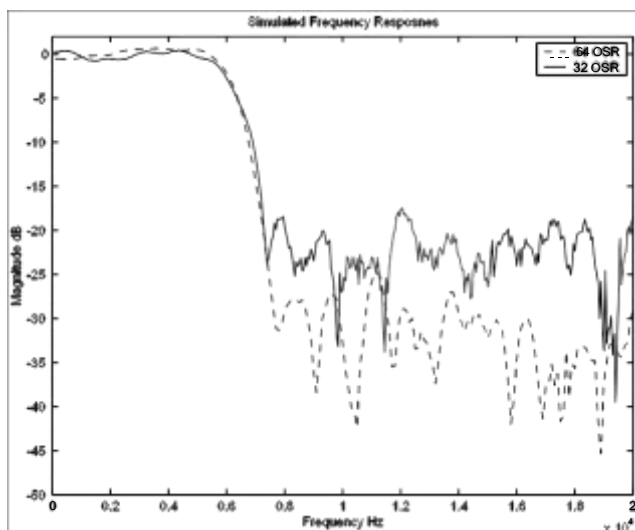
The Remez exchange method was used to produce a target impulse response  $G(z)$ , which was then used to create the taps. Since oversampling is necessary for the noise-shaping of the  $m$ , it was used to generate the desired response. [1] The maximum input value of the  $m$  was then used to normalize this answer. While scaling the ternary filter does introduce a constant gain, it significantly enhances the filter's stopband attenuation. At the ternary filter's output, the gain may be subtracted by multiplying the inverse gain by it. The filter construction seen in Figure 1 demonstrates the inverse gain factor  $a$ .

For the ternary taps to be usable, the digital  $m$ 's used to modulate them must meet specific criteria. Such features need for a ternary quantizer. The modulator's signal transfer function (STF) should not significantly alter the desired impulse response throughout the relevant frequency range. The lowpass target impulse response was re-quantized using a second-order double-loop modulator; the curious reader is directed to [8] for further information.

Since the ternary FIR filter produces multi-bit output, which is inefficient for hardware implementation, a recursive filter was employed to re-quantize the output to single-bit. This iterative structure uses a  $m$  to do remodulation. Within the recursive filter, the  $m$  is used as a delay component. The usage of a  $m$  simplifies the code and also converts the output back to a single-bit format.

Where  $f_s$  is the sampling frequency and  $2\pi f / f_s$  is the normalized radian frequency. For further information on the frequency response, please see [8].

Typical speech filter settings were used in a simulation of the stated lowpass filter. We used a 5-kilohertz 3-dB cutoff and an 8-kilohertz 8-kilohertz stopband. Figure 3 displays the outcome of running a simulation of the system with 64 and 32 oversampling ratios (OSR).



**Figure 3:** Frequency response for overlapping ratios of 32 and 64.

### 3 FILTER WITH A SINGLE BANDPASS BIT

Keeping with the theme of single-bit filtering methods, we now go from lowpass to bandpass operation. Bandpass single-bit ternary filter schematic shown in Figure 4. Readers should take notice that unlike the lowpass single-bit filter, the remodulation process makes use of a bandpass  $m$  not a recursive  $m$ -based filter. The design of the one-bit bandpass filter is simplified as a result. A further advantage of the bandpass single-bit filter over the lowpass one is the ease with which its stability can be analyzed. This is due to the fact that the single-bit lowpass filter's recursive loop might exhibit stability problems of its own.

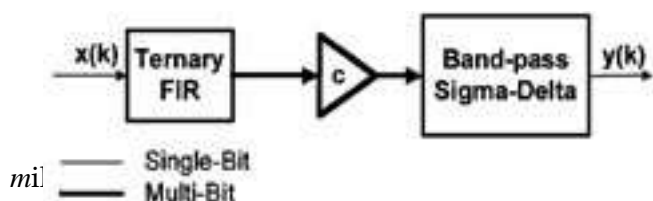
The ternary taps were generated using a ternary quantizer in lieu of a quantizer of order  $m$ . To maximize the modulator's SQNR, a ternary quantizer with equally spaced thresholds was utilized.<sup>2</sup>

This 8-order bandpass  $m$ 's NTF specifies the filtering response on the input signal, while its STF specifies the bandwidth  $f_b$ . The following equations illustrate the 8th-order  $m$  STF and NTF, where  $a_i$  and  $m_i$  are constants that determine the pole-zero positions.

$$STF = \frac{A(z)}{B(z)} \quad (3)$$

where

$$A(z) = a_4 z^{-2} + (a_3 + 3a_4 - m_1 a_4 - m_3 a_4) z^{-4} + (a_2 + 2a_3 + 3a_4 - m_2 (a_1 + a_3 - m_4) - 2m_3 a_4) z^{-6} + (a_1 + a_2 + a_3 + a_4 - m_1 a_2 - m_3 a_4) z^{-8}$$



structure contains an efficient ternary filter. The ternary filter is cascaded with a bandpass  $\Sigma\Delta m$ . Here the  $\Sigma\Delta m$  is used to remodulate the multi-bit signal generated by the ternary filter back to a single-bit

Bandpass filters with a center frequency of  $f_s/4$  have no poles or zeroes that are odd powers of  $z$  in the noise transfer function (NTF) or the spectral transfer function (STF). Bandpass modulators with other than  $f_s/4$  center frequencies have been created. The ternary taps are encoded by the system function of the 8th-order bandpass  $m$  in the  $z$ -domain, which is given by:

$$A(z) > C(z) > E(z) > Y(z).$$

are uncommon and often more difficult to implementation. But if we use a bandpass  $m$

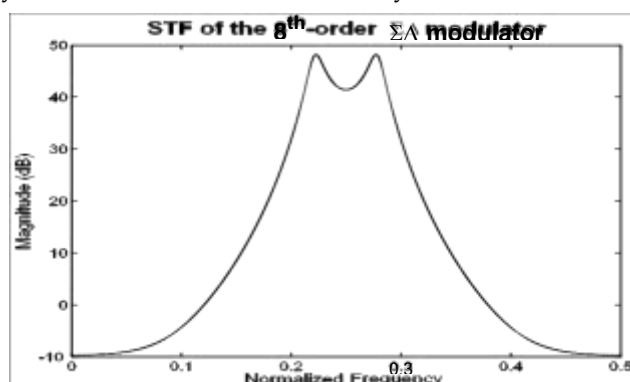
(5)

where  $E(z)$  is the representation of the quantization noise in the  $z$ -domain and  $X(z)$  is the representation of the input signal in the  $z$ -domain.

Figure 5 and Figure 6 show the signal and noise frequency responses of the 8th-order bandpass  $m$  used to encode the ternary taps. The constants' values were established in<sup>13</sup>.

"A one-bit digital bandpass filter with a narrow passband" - Thompson, Hussain, and O'Shea, Chapter 5 and work best with integers in the  $2^n$  range. The parameters utilized were:  $(a)_2-6$ ,  $(a)_2-3$ ,  $(a)_0.5$ ,  $(a)_-1$ ,  $(m)_12-6$ ,  $(m)_0$ , and  $(m)_0$ .

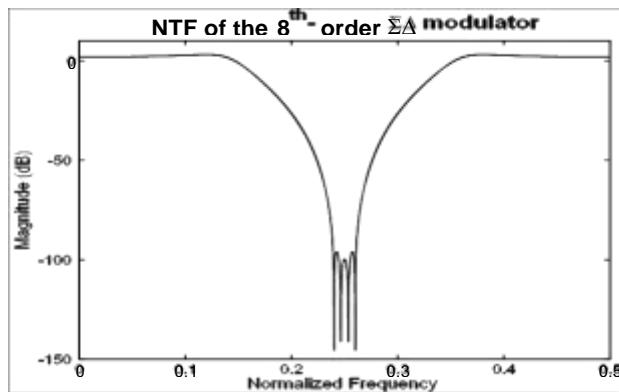
NTF in Figure 7 has the same shape as the theoretical NTF in Figure 6, particularly in the noise null at the center frequency where the three lobes can clearly be seen.



**Figure 5:** STF of the 8<sup>th</sup>-order bandpass  $\Sigma\Delta$  ternary modulator.

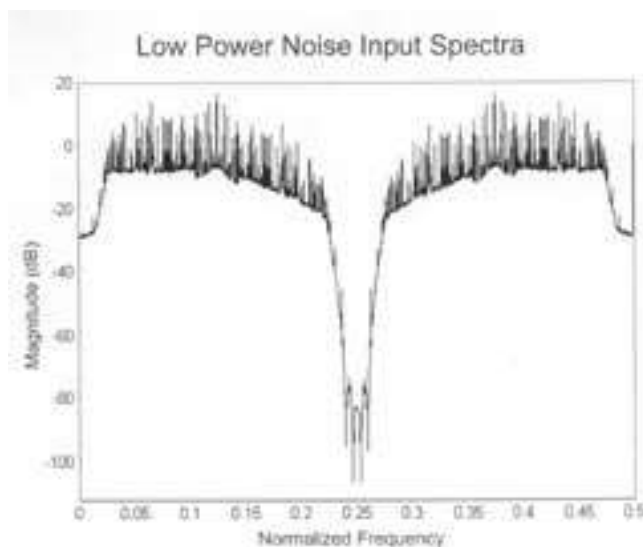
The 8th order bandpass m STF features peaks on both the low and high sides of the central frequency. Due to their proximity to the shaped quantization noise (see Figure 6), these peaks have negligible effects on the modulated ternary taps. The flat portion of the STF coincides with the region of the NTF where quantization noise attenuation is

greatest. As a result, input signals in this frequency range won't be distort.



**Figure 6:** NTF of the 8<sup>th</sup>-order modulator.

To verify the NTF of the 8th-order bandpass  $\Sigma\Delta$ m, a simulation was undertaken where a low-power white noise signal was input to the 8<sup>th</sup>-order bandpass



**Figure 7:** Simulated NTF of the 8<sup>th</sup>-order ternary  $\Sigma\Delta$ m .

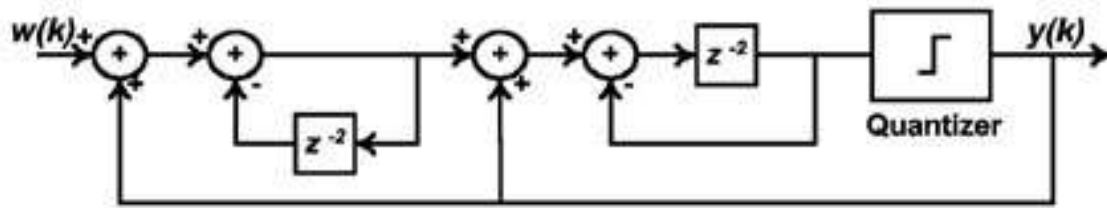
Multiplying the filter output (before to remodulation) by the inverse gain, shown by c in Figure 4, cancels out the gain created by the ternary filter. The input signal's amplitude is lowered by this inverse gain factor so as not to overwhelm the m selected for re-quantization.

The filtered signal must be re-quantized into the single-bit domain, which necessitates the use of a m with a signal-to-quantization noise ratio (SQNR) larger than the attenuation of the ternary filter's stopband. It also needs a wider bandwidth than the ternary filter's passband, otherwise the stopband attenuation will be compromised due to quantization noise. Keeping the hardware implementation as basic and efficient as feasible in order to preserve efficient single-bit implementations is an additional criteria, although a less rigorous one.

Bandpass double-loop m modulators meet these specifications.<sup>12</sup> The framework for this

Figure 8 displays the value of m. The modulator was built by applying the  $z$ - $z^{-2}$  transform on the lowpass form, converting it into a bandpass filter. A bandpass modulator with a core frequency of  $f_s/4$  is thus produced. The m satisfies these conditions despite being six orders lower than the m utilized to encode the ternary taps. This is accomplished in a framework that uses just delays, a summer, and a quantizer (no multiplication is required).





**Figure 8:** Block diagram of the double loop bandpass  $\Sigma\Delta_m$ .

The z-domain system function of the bandpass double-loop  $\Sigma\Delta_m$  is given by:

stable. The stability of the double-loop bandpass

$\Sigma\Delta_m$  is studied in<sup>12</sup>, where it was found to contain  $Y_{DL\Delta}$

$$(z) = W(z)z^{-2} + N(z)(1 + 2z^{-2} + z^{-4})$$

limit-cycles identical to those in the double-loop lowpass  $\Sigma\Delta_m$ , which is generally stable.<sup>1</sup>

Assuming the single-bit filtering system is linear and time-invariant, the larger system may be described as follows:

quantization noise z-domain transform. It is clear from the phrase  $(1+2z^{-2}+z^{-4})$  that the modulator has bandpass noise-shaping capabilities. From equation (6) we can get the NTF and STF of the bandpass m:

When it comes to remodulation, an IIR loop is not used, in contrast to the single-bit lowpass filter discussed above. Since the ternary filter is intrinsically unstable, the stability of the single-bit bandpass filter is totally reliant on the bandpass double-loop m.

The ramification of considering the system to be linear-time-invariant is that the quantizer is considered to be a linear element. Although this is generally considered a simplistic model, it allows simplifying of the inherently non-linear quantizer. This linearization of the quantizer has also found to provide insight into the stability of  $\Sigma\Delta_m$ 's.<sup>1</sup>

The final system function demonstrates that the ternary filter, the NTF of the double-loop m, or both will attenuate the terms containing  $C(z)/D(z)$  or  $(1+2z^{-2}+z^{-4})$ . Only the product of the encoded impulse-responses STF and the double-loop m's STF,  $X(z)(G(z)A(z)/B(z))z^{-2}$ , remains. Since  $G(ej)$  is the desired frequency response and  $A(z)/B(z)$  is the STF of the 8th-order bandpass m, the final frequency response will have the same shape as  $G(ej)$ . This is because the target impulse-response accomplishes the filtering, and the STF of the 8th-order bandpass m is relatively flat in the area of the quantization noise.

#### 4 CONCLUSIONS AND RESULTS

Simulations were run to demonstrate the effectiveness of the proposed single-bit bandpass filter. White

"A one-bit digital bandpass filter with a narrow passband" As cited in Thompson, Hussain, and O'Shea (2007:7)

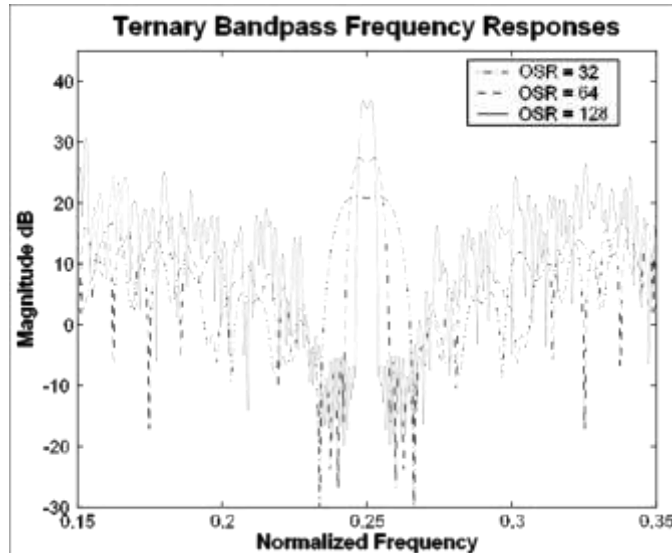
Bandpass single-bit filtering was used on Gaussian noise. In order to achieve 8192 data points, plus the length of the ternary filter, the signal length has to be modified. After passing the data through the ternary filter, the transient was eliminated, leaving just the filtered noise. Fast Fourier transform (FFT) was used to determine the frequency content of the filtered noise. Finally, the FFT was calculated as an average across several iterations of the simulation. Based on the weighted average of the FFT, we can make an educated guess as to the frequency response of the one-bit filter. A rough estimate of the filtering system's frequency response may be seen in the average of the simulated FFTs, as the bandlimited white noise used in the simulation covers all frequencies up to  $f_s/2$ .

Oversampling ratios of 32, 64, and 128 required corresponding ternary filters. The remez exchange method was used to generate the three frequency responses  $G(ej)$ . For all three OSRs, we decided on a passband width of 4 kHz. In this case, the Nyquist rate was set at 8 kilohertz. The Nyquist rate multiplied by the OSR was used to get the sample frequency  $f_s$ . The bandwidth for the transition from the stopband was fixed at 2 kHz. A common speech filter might look like this.

We used a value close to the ternary filter's maximum attenuation setting for the stopband attenuation. This is an iterative procedure. The first step is to create the coefficients using multiple bits. The ternary representation is achieved by scaling and modulating these factors. To begin, a series of taps with a stopband attenuation beyond the capabilities of the ternary filter is generated. The ternary taps were generated by encoding the multi-bit target impulse response with an 8th-order bandpass m. Figure 10 displays the desired frequency responses for each oversampling ratio. For OSR, the stopband attenuations in the desired frequency responses were 29, 39, and 47 dB.

equals 32, 64, and 128.

Absolute maximum amplitude of around 0.80.01 was achieved by scaling the desired impulse-response amplitudes. The stopband attenuation was maximized using this scaling factor. It was also discovered in earlier research that the target taps scaled.8 In order to encode the ternary taps optimally, the authors assume that the SQNR of the specific modulator utilized is connected to optimum scaling. Since the input SQNR curves vary throughout  $m$ , the best scaling factor must also vary across  $m$ . More research is needed on this particular facet of filter tap encoding. Figure 11 depicts the encoded ternary frequency responses as taps.



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“A single-bit narrow-band bandpass digital filter” - Thompson, Hussain & O’Shea

## CONCLUSION

## Bandpass

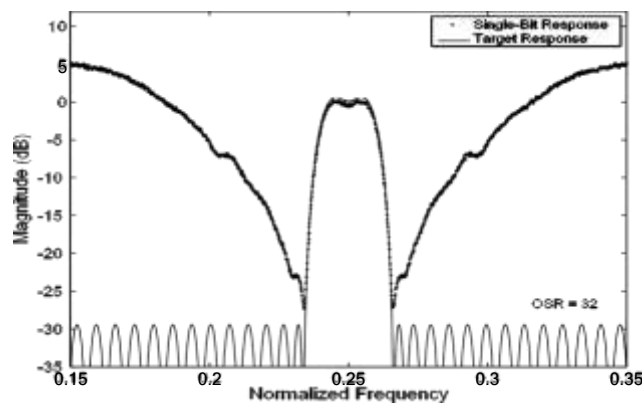


Figure 12:

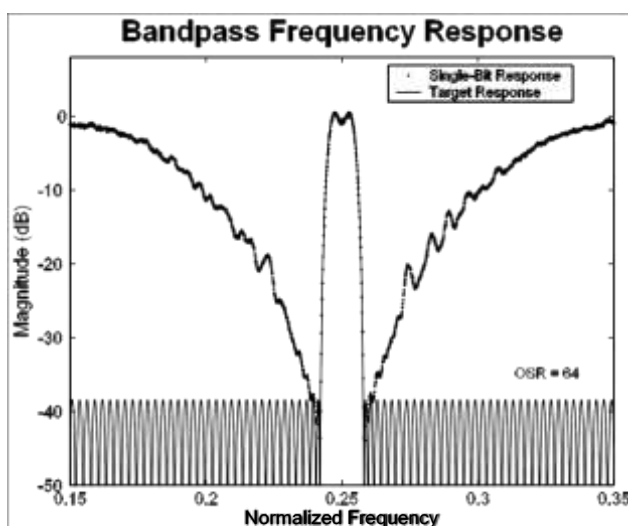


Figure 13:

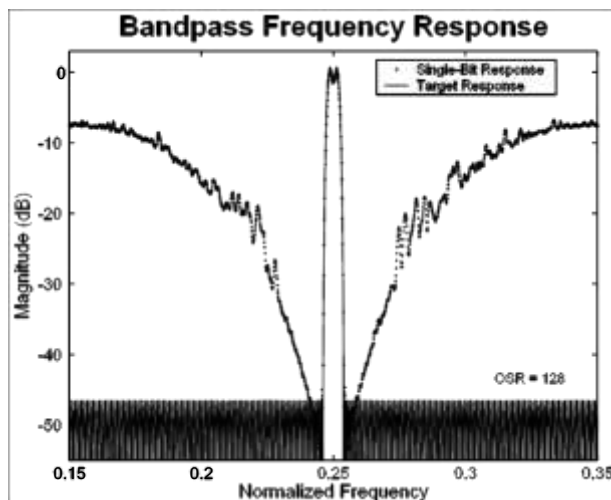
### Frequency Response

Target and final frequency response at  $OSR = 64$ , with 8192 point windows FFT.

We introduce a low-pass, one-bit filter. Since the lowpass case was already covered, the bandpass case was added to the single-bit filter. In the bandpass instance, there is no internal IIR filter, which simplifies the filter's construction.

although a more straightforward remodulating version of  $m$  is utilized. We show the outcomes for a standard voice filter with oversampling ratios of 32, 64, and 128. As one would guess, the ternary filter's stopband attenuation improves as the oversampling factor rises.

Figure 14: Target and final frequency response at  $OSR = 128$ , with 8192 point windows FFT.



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This is because, at larger OSRs, the remodulating 8th-order bandpass  $m$  is better able to shape noise. Since the remodulating double-loop bandpass  $m$  noise nulling zone becomes larger with OSR, greater OSR also gives a broader bandwidth. This increased bandwidth is significant since the ternary filter's stopband attenuation is diminished (hidden in quantization noise) if the OSR is not high enough.

The design of this one-bit filter still need further study. In the bandpass situation, using an IIR filter structure (like that used in the lowpass example) may enhance implementation efficiency and stopband attenuation. The stability of such a building would need to be studied, of course. Finding the best setting for the scaling factor used in ternary encoding requires some digging. Before encoding filter coefficients, knowing the appropriate scaling factor for maximal stopband attenuation is helpful.

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